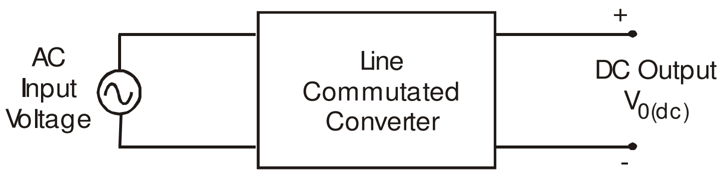
UNIT-4

Controlled Rectifiers

4.1 Line Commutated AC to DC converters



- Type of input: Fixed voltage, fixed frequency ac power supply.
- Type of output: Variable dc output voltage
- Type of commutation: Natural / AC line commutation

4.1.1Different types of Line Commutated Converters

- AC to DC Converters (Phase controlled rectifiers)
- AC to AC converters (AC voltage controllers)
- AC to AC converters (Cyclo converters) at low output frequency

4.1.2 Differences Between Diode Rectifiers & Phase Controlled Rectifiers

- The diode rectifiers are referred to as uncontrolled rectifiers .
- The diode rectifiers give a fixed dc output voltage .
- Each diode conducts for one half cycle.
- Diode conduction angle = 180^0 or π radians.
- We cannot control the dc output voltage or the average dc load current in a diode rectifier circuit

Single phase half wave diode rectifier gives an

Average dc output voltage $V_{O(dc)} = \frac{V_m}{\pi}$

Single phase full wave diode rectifier gives an

Average dc output voltage $V_{O(dc)} = \frac{2V_m}{\pi}$

4.2 Applications of Phase Controlled Rectifiers

- DC motor control in steel mills, paper and textile mills employing dc motor drives.
- AC fed traction system using dc traction motor.
- Electro-chemical and electro-metallurgical processes.
- Magnet power supplies.

- Portable hand tool drives

4.3 Classification of Phase Controlled Rectifiers

- Single Phase Controlled Rectifiers.
- Three Phase Controlled Rectifiers

4.3.1 Different types of Single Phase Controlled Rectifiers.

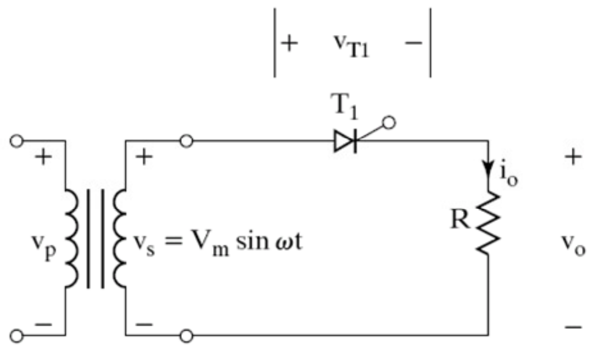
- Half wave controlled rectifiers.
- Full wave controlled rectifiers.
- Using a center tapped transformer.
- Full wave bridge circuit.
- Semi converter.
- Full converter.

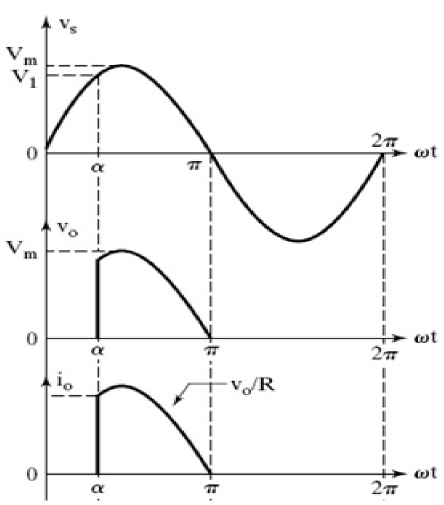
4.3.2 Different Types of Three Phase Controlled Rectifiers

- Half wave controlled rectifiers.
- Full wave controlled rectifiers.
- Semi converter (half controlled bridge converter).
- Full converter (fully controlled bridge converter).

4.4 Principle of Phase Controlled Rectifier Operation

Single Phase Half-Wave Thyristor Converter with a Resistive Load





Equations:

$v_s = V_m \sin \omega t =$ i/p ac supply voltage

$V_m =$ max. value of i/p ac supply voltage

$V_S = \frac{V_m}{\sqrt{2}} =$ RMS value of i/p ac supply voltage

ω α $v_O = v_L =$ output voltage across the load

When the thyristor is triggered at $t =$

$v_O = v_L = V_m \sin \omega t;\ \omega t = \alpha$ to π

$i_O = i_L = \frac{v_O}{R} =$ Load current; $\omega t = \alpha$ to π

$i_O = i_L = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t; \omega t = \alpha$ to π

Where $I_m = \frac{V_m}{R} =$ max. value of load current

4.4.1 To Derive an Expression for the Average (DC) Output Voltage across the Load

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi} \int_0^{2\pi} v_O . d(\omega t);$$

$$v_O = V_m \sin \omega t \ for\ \ \omega t = \alpha \ to\ \pi$$

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi} \int_\alpha^{\pi} V_m \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = \frac{1}{2\pi} \int_\alpha^{\pi} V_m \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi} \int_\alpha^{\pi} \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi} \left[-\cos \omega t \Big/_{\alpha}^{\pi} \right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi} [-\cos \pi + \cos \alpha];\ \ \cos \pi = -1$$

$$V_{O(dc)} = \frac{V_m}{2\pi} [1 + \cos \alpha]\ ;\ V_m = \sqrt{2} V_S$$

Maximum average (dc) o/p
voltage is obtained when $\alpha = 0$
and the maximum dc output voltage

$$V_{dc(\max)} = V_{dm} = \frac{V_m}{2\pi}(1+\cos 0); \ \cos(0) = 1$$

$$\therefore V_{dc(\max)} = V_{dm} = \frac{V_m}{\pi}$$

$$V_{O(dc)} = \frac{V_m}{2\pi}[1+\cos\alpha] \ ; V_m = \sqrt{2}V_S$$

The average dc output voltage can be varied by varying the trigger angle α from 0 to a maximum of 180^0 (π radians)

We can plot the control characteristic $(V_{O(dc)} \text{ vs } \alpha)$ by using the equation for $V_{O(dc)}$

4.5 Control Characteristic of Single Phase Half Wave Phase Controlled Rectifier with Resistive Load

The average dc output voltage is given by the expression

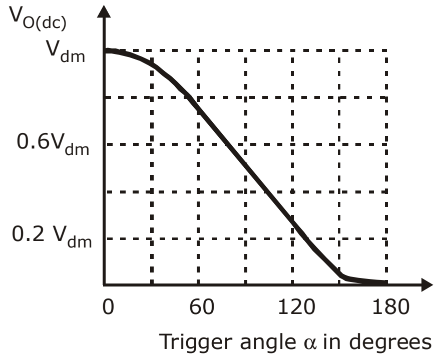
$$V_{O(dc)} = \frac{V_m}{2\pi}[1+\cos\alpha]$$

We can obtain the control characteristic by plotting the expression for the dc output voltage as a function of trigger angle α

Trigger angle α in degrees	$V_{O(dc)}$	**%**
0	$V_{dm} = \frac{V_m}{\pi}$	100% V_{dm}
30^0	0.933 V_{dm}	93.3 % V_{dm}
60^0	0.75 V_{dm}	75 % V_{dm}
90^0	0.5 V_{dm}	50 % V_{dm}
120^0	0.25 V_{dm}	25 % V_{dm}
150^0	0.06698 V_{dm}	6.69 % V_{dm}
180^0	0	0

$$V_{dm} = \frac{V_m}{\pi} = V_{dc(\max)}$$

4.5.1 Control Characteristic



Normalizing the dc output voltage with respect to V_{dm}, the Normalized output voltage

$$V_n = \frac{V_{dc}}{V_{dm}} = \frac{\frac{V_m}{2\pi}(1+\cos\alpha)}{\frac{V_m}{\pi}}$$

$$V_n = \frac{V_{dc}}{V_{dm}} = \frac{1}{2}(1+\cos\alpha) = V_{dcn}$$

4.5.2 To Derive an Expression for the RMS Value of Output Voltage of a Single Phase Half Wave Controlled Rectifier with Resistive Load

The RMS output voltage is given by

$$V_{O(RMS)} = \left[\frac{1}{2\pi}\int_0^{2\pi} v_O^2.d(\omega t)\right]$$

Output voltage $v_O = V_m \sin\omega t$; for $\omega t = \alpha$ to π

$$V_{O(RMS)} = \left[\frac{1}{2\pi}\int_\alpha^{\pi} V_m^2 \sin^2\omega t.d(\omega t)\right]^{\frac{1}{2}}$$

By substituting $\sin^2\omega t = \frac{1-\cos 2\omega t}{2}$, we get

$$V_{O(RMS)} = \left[\frac{1}{2\pi}\int_\alpha^{\pi} V_m^2 \frac{(1-\cos 2\omega t)}{2}.d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi}\int_\alpha^{\pi} (1-\cos 2\omega t)\,d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi}\left\{\int_\alpha^{\pi} d(\omega t) - \int_\alpha^{\pi}\cos 2\omega t\,d(\omega t)\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left\{(\omega t)\Big/_{\alpha}^{\pi} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\pi}\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left((\pi-\alpha) - \frac{(\sin 2\pi - \sin 2\alpha)}{2}\right)\right]$$; $\sin 2\pi = 0$

4.5.3 Performance Parameters of Phase Controlled Rectifiers

Output dc power (avg. or dc o/p power delivered to the load)

$P_{O(dc)} = V_{O(dc)} \times I_{O(dc)}$; *i.e.*, $P_{dc} = V_{dc} \times I_{dc}$

Where

$V_{O(dc)} = V_{dc} =$ avg./ dc value of o/p voltage.

$I_{O(dc)} = I_{dc} =$ avg./dc value of o/p current

Output ac power

$$P_{O(ac)} = V_{O(RMS)} \times I_{O(RMS)}$$

Efficiency of Rectification (Rectification Ratio)

Efficiency $\eta = \frac{P_{O(dc)}}{P_{O(ac)}}$; % Efficiency $\eta = \frac{P_{O(dc)}}{P_{O(ac)}} \times 100$

The o/p voltage consists of two components

The dc component $V_{O(dc)}$

The ac /ripple component $V_{ac} = V_{r(rms)}$

Output ac power

$$P_{O(ac)} = V_{O(RMS)} \times I_{O(RMS)}$$

Efficiency of Rectification (Rectification Ratio)

Efficiency $\eta = \frac{P_{O(dc)}}{P_{O(ac)}}$; % Efficiency $\eta = \frac{P_{O(dc)}}{P_{O(ac)}} \times 100$

The o/p voltage consists of two components

The dc component $V_{O(dc)}$

The ac /ripple component $V_{ac} = V_{r(rms)}$

4.5.4 The Ripple Factor (RF) w.r.t output voltage waveform

$$r_v = RF = \frac{V_{r(rms)}}{V_{O(dc)}} = \frac{V_{ac}}{V_{dc}}$$

$$r_v = \frac{\sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}}{V_{O(dc)}} = \sqrt{\left[\frac{V_{O(RMS)}}{V_{O(dc)}}\right]^2 - 1}$$

$$r_v = \sqrt{FF^2 - 1}$$

Current Ripple Factor $r_i = \frac{I_{r(rms)}}{I_{O(dc)}} = \frac{I_{ac}}{I_{dc}}$

Where $I_{r(rms)} = I_{ac} = \sqrt{I_{O(RMS)}^2 - I_{O(dc)}^2}$

$V_{r(pp)} =$ peak to peak ac ripple output voltage

$$V_{r(pp)} = V_{O(\max)} - V_{O(\min)}$$

$I_{r(pp)} =$ peak to peak ac ripple load current

$$I_{r(pp)} = I_{O(\max)} - I_{O(\min)}$$

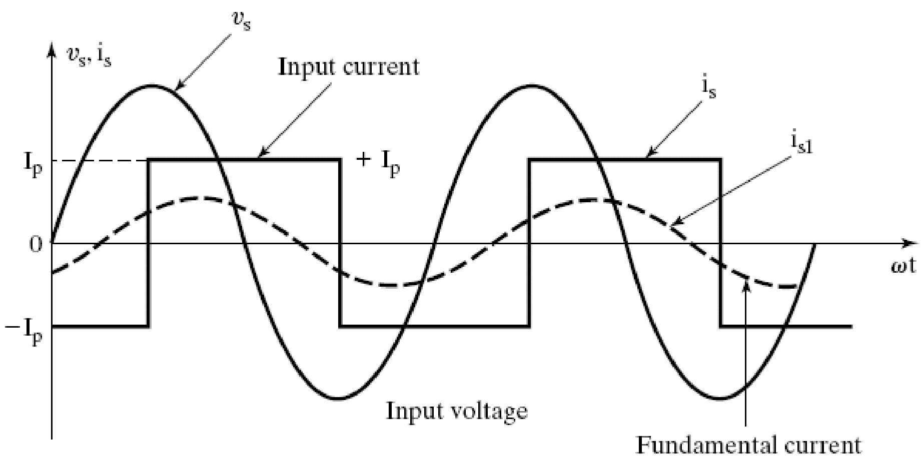
Transformer Utilization Factor (TUF)

$$TUF = \frac{P_{O(dc)}}{V_S \times I_S}$$

Where

$V_S =$ RMS supply (secondary) voltage

$I_S =$ RMS supply (secondary) current



Where

$v_S =$ Supply voltage at the transformer secondary side

$i_S =$ i/p supply current

(transformer secondary winding current)

$i_{S1} =$ Fundamental component of the i/p supply current

= Displacement angle (phase angle)

For an RL load

= Displacement angle = Load impedance angle

$$\therefore \quad \phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \text{ for an RL load}$$

Displacement Factor (DF) or

Fundamental Power Factor

$$DF = Cos\phi$$

Harmonic Factor (HF) or

Total Harmonic Distortion Factor ; THD

$$HF = \left[\frac{I_S^2 - I_{S1}^2}{I_{S1}^2}\right]^{\frac{1}{2}} = \left[\left(\frac{I_S}{I_{S1}}\right)^2 - 1\right]^{\frac{1}{2}}$$

Where

I_S = RMS value of input supply current.

I_{S1} = RMS value of fundamental component of the i/p supply current.

Input Power Factor (PF)

$$PF = \frac{V_S I_{S1}}{V_S I_S}\cos\phi = \frac{I_{S1}}{I_S}\cos\phi$$

The Crest Factor (CF)

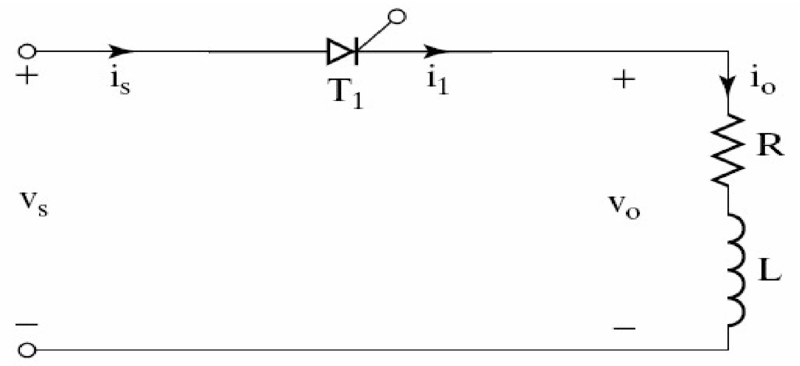
$$CF = \frac{I_{S(peak)}}{I_S} = \frac{\text{Peak input supply current}}{\text{RMS input supply current}}$$

For an Ideal Controlled Rectifier

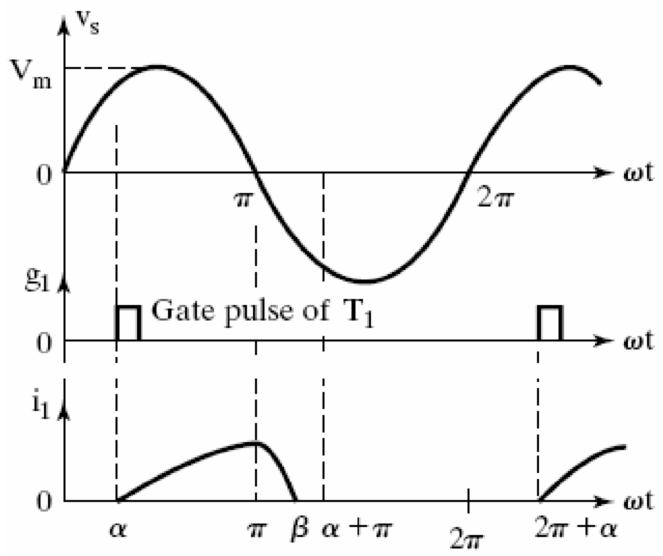
$FF = 1$; $\eta = 100\%$; $V_{ac} = V_{r(rms)} = 0$; $TUF = 1$;

$RF = r_v = 0$; $HF = THD = 0$; $PF = DPF = 1$

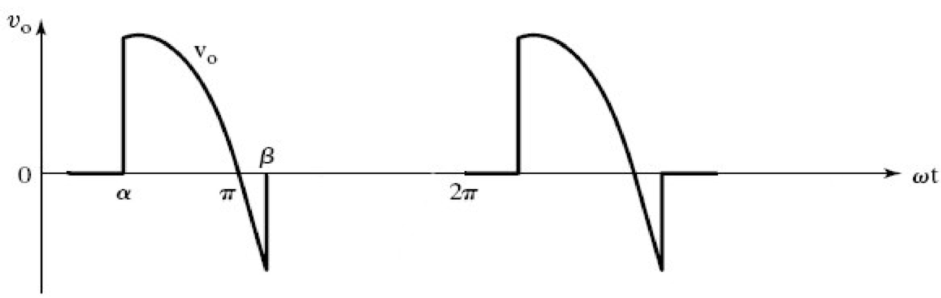
4.5.5 Single Phase Half Wave Controlled Rectifier with an RL Load



Input Supply Voltage (Vs) & Thyristor (Output) Current Waveforms



Output (Load) Voltage Waveform



4.5.6 To derive an expression for the output (Load) current, during ωt = α to β when

thyristor T_1 conducts

Assuming T_1 is triggered $\omega t = \alpha$,
we can write the equation,

$$L\left(\frac{di_O}{dt}\right) + Ri_O = V_m \sin \quad t \; ; \quad \le \quad t \le$$

ω φ τ

General expression for the output current,

$$i_O = \frac{V_m}{Z}\sin(\quad t - \quad) + A_1 e^{\frac{-t}{}}$$

ω

$V_m = \sqrt{2}V_S$ = maximum supply voltage.

$Z = \sqrt{R^2 + (\quad L)^2}$ =Load impedance.

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ = Load impedance angle.

$\tau = \frac{L}{R}$ = Load circuit time constant.

$\therefore$ general expression for the output load current

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-R}{L}t}$$

Constant A_1 is calculated from

initial condition $i_O = 0$ at $\omega t = \alpha$; t=$\left(\frac{\alpha}{\omega}\right)$

$$i_O = 0 = \frac{V_m}{Z}\sin(\alpha - \phi) + A_1 e^{\frac{-R}{L}t}$$

$$\therefore \quad A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin(\alpha - \phi)$$

We get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

Substituting the value of constant A_1 in the general expression for i_O

$$i_O = \frac{V_m}{Z}\sin(\quad t - \quad) + e^{\frac{-R}{\omega L}(\omega t - \alpha)}\left[\frac{-V_m}{Z}\sin(\quad - \quad)\right]$$

∴ we obtain the final expression for the inductive load current

ω φ α φ

$$i_O = \frac{V_m}{Z}\left[\sin(\quad t - \quad) - \sin(\quad - \quad)e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right];$$

Where $\alpha \le \omega t \le \beta$

ω β

Extinction angle can be calculated by using the condition that $i_O = 0$ at $t =$

$$i_O = \frac{V_m}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right] = 0$$

$$\therefore \sin(\beta - \phi) = e^{\frac{-R}{\omega L}(\beta - \alpha)} \times \sin(\alpha - \phi)$$

β can be calculated by solving the above eqn.

4.5.7 To Derive an Expression for Average (DC) Load Voltage of a Single Half Wave Controlled Rectifier with RL Load

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\int_0^{2\pi} v_O.d(\omega t)$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_0^{\alpha} v_O.d(\omega t) + \int_{\alpha}^{\beta} v_O.d(\omega t) + \int_{\beta}^{2\pi} v_O.d(\omega t)\right]$$

$v_O = 0$ for $\omega t = 0$ to α & for $\omega t = \beta$ to 2π

$$\therefore V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} v_O.d(\omega t)\right];$$

$v_O = V_m \sin \omega t$ for $\omega t = \alpha$ to β

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} V_m \sin \omega t.d(\omega t)\right]$$

$$V_{O(dc)} = V_L = \frac{V_m}{2\pi}\left[-\cos \omega t \Big/_{\alpha}^{\beta}\right]$$

$$V_{O(dc)} = V_L = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

$$\therefore V_{O(dc)} = V_L = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

Effect of Load Inductance on the Output

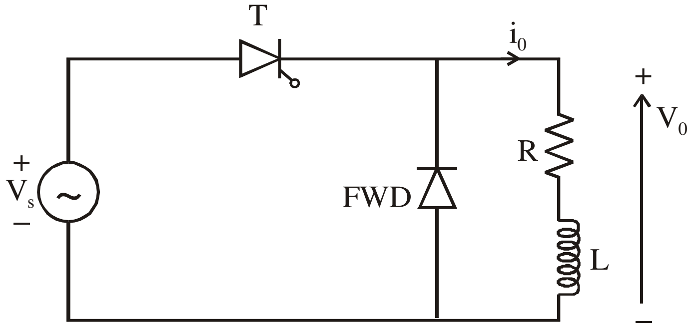
During the period ωt = Π to β the instantaneous output voltage is negative and this reduces the average or the dc output voltage when compared to a purely resistive load.

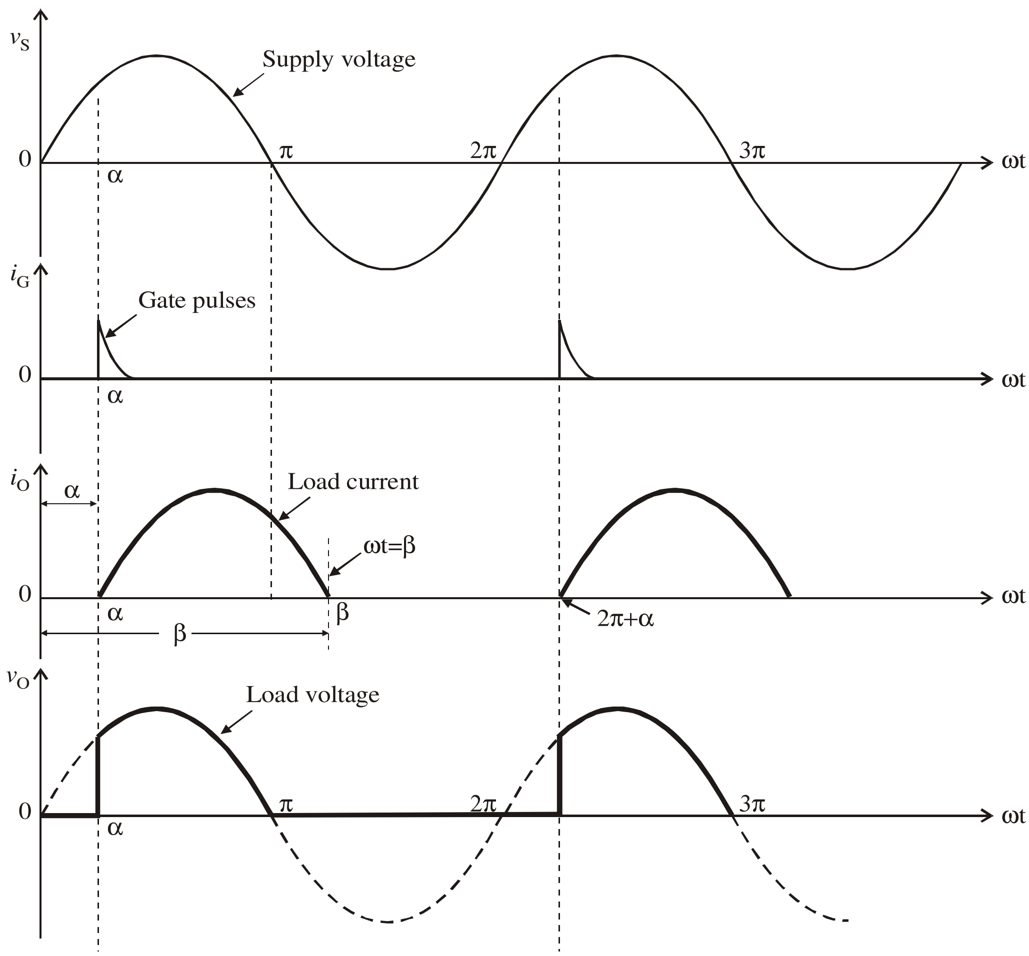
4.5.8 Average DC Load Current

π

$$I_{O(dc)} = I_{L(Avg)} = \frac{V_{O(dc)}}{R_L} = \frac{V_m}{2\ \ R_L}\left(\cos\ \ \ -\cos\ \ \ \right)$$

4.5.9 Single Phase Half Wave Controlled Rectifier with RL Load & Free Wheeling Diode





The average output voltage

$V_{dc} = \frac{V_m}{2\pi}[1+\cos\alpha]$ which is the same as that of a purely resistive load.

The following points are to be noted

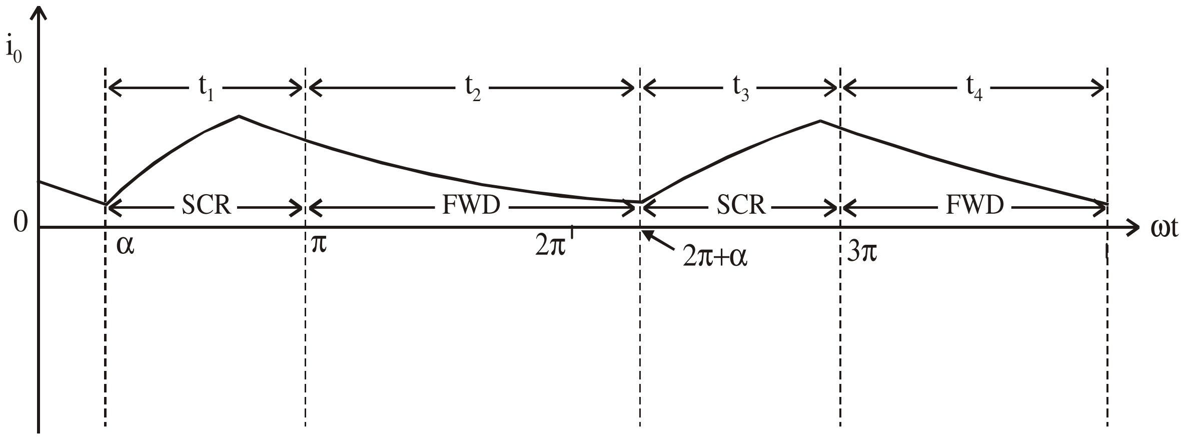
For low value of inductance, the load current tends to become discontinuous.

During the period α to π the load current is carried by the SCR.

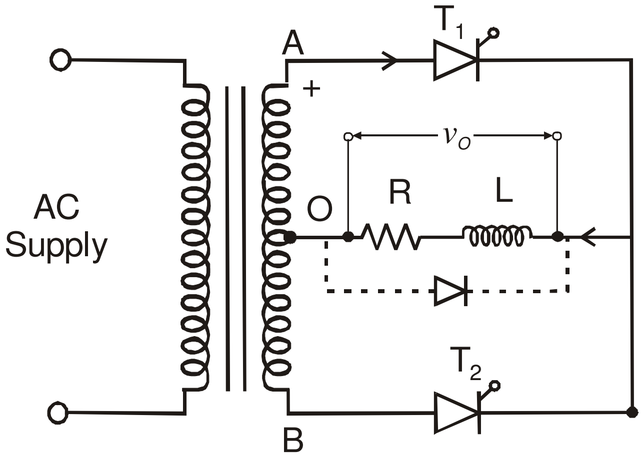
During the period π to β load current is carried by the free wheeling diode.

The value of β depends on the value of R and L and the forward resistance of the FWD.

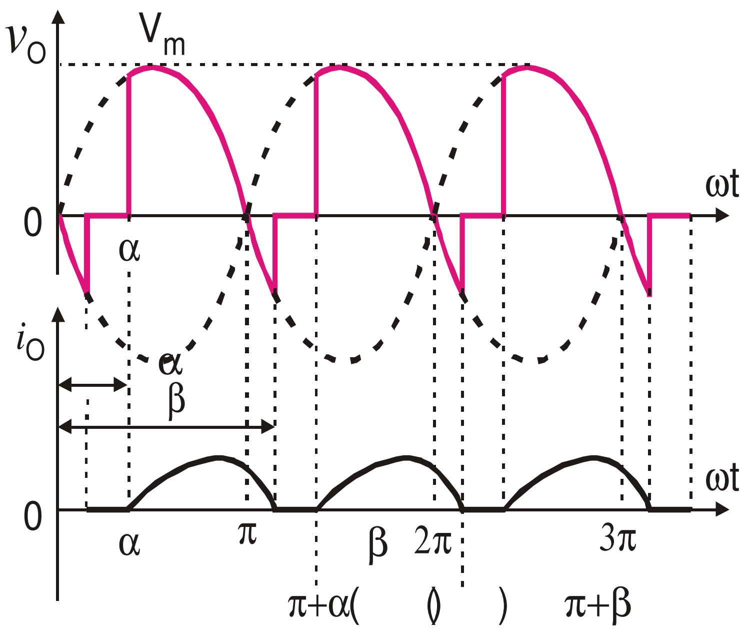
For Large Load Inductance the load current does not reach zero, & we obtain continuous load current.



4.6 Single Phase Full Wave Controlled Rectifier Using A Center Tapped Transformer



4.6.1 Discontinuous Load Current Operation without FWD for π <β< (π+α)



(i) To derive an expression for the output (load) current, during ωt = α to β when thyristor T$_1$ conducts

Assuming T_1 is triggered $\omega t = \alpha$,

we can write the equation,

$$L\left(\frac{di_O}{dt}\right)+Ri_O=V_m\sin\omega t;\quad \alpha\le\omega t\le\beta$$

General expression for the output current,

$$i_O=\frac{V_m}{Z}\sin(\omega t-\phi)+A_1e^{\frac{-t}{\tau}}$$

Constant A_1 is calculated from

initial condition $i_O = 0$ at $\omega t = \alpha$; t=$\left(\frac{\alpha}{\omega}\right)$

$$i_O = 0 = \frac{V_m}{Z}\sin(\alpha - \phi) + A_1 e^{\frac{-R}{L}t}$$

$$\therefore \quad A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin(\alpha - \phi)$$

We get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

ω

$V_m = \sqrt{2}V_S$ = maximum supply voltage.

$Z = \sqrt{R^2 + (\ \ L)^2}$ =Load impedance.

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ = Load impedance angle.

$\tau = \frac{L}{R}$ = Load circuit time constant.

$\therefore$ general expression for the output load current

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-R}{L}t}$$

Substituting the value of constant A_1 in the general expression for i_O

$$i_O = \frac{V_m}{Z}\sin(\ \ t - \ \ ) + e^{\frac{-R}{\omega L}(\omega t - \alpha)}\left[\frac{-V_m}{Z}\sin(\ \ - \ \ )\right]$$

$\therefore$ we obtain the final expression for the inductive load current

ω ϕ α ϕ

$$i_O = \frac{V_m}{Z}\left[\sin(\ \ t - \ \ ) - \sin(\ \ - \ \ )e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right];$$

Where $\alpha \le \omega t \le \beta$

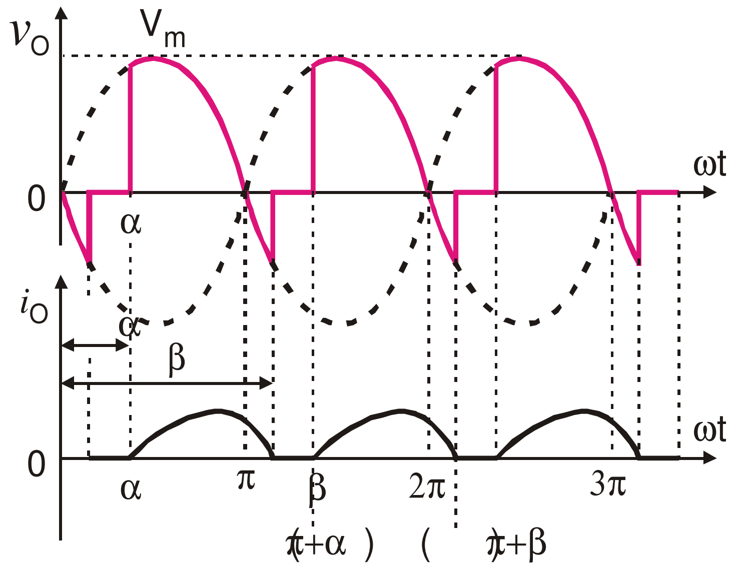
Extinction angle ω β can be calculated by using the condition that $i_O = 0$ $at \quad t =$

$$i_O = \frac{V_m}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right] = 0$$

$$\therefore \sin(\beta - \phi) = e^{\frac{-R}{\omega L}(\beta - \alpha)} \times \sin(\alpha - \phi)$$

β can be calculated by solving the above eqn.

(ii) To Derive an Expression for the DC Output Voltage of A Single Phase Full Wave Controlled Rectifier with RL Load *(Without FWD)*



$$V_{O(dc)} = V_{dc} = \frac{1}{\pi}\int_{\omega t=\alpha}^{\beta} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi}\left[\int_{\alpha}^{\beta} V_m \sin \omega t . d(\omega t)\right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[-\cos \omega t \Big/_{\alpha}^{\beta}\right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}(\cos\alpha - \cos\beta)$$

When the load inductance is negligible (i.e., $L \approx 0$)

Extinction angle $\beta = \pi$ radians

Hence the average or dc output voltage for R load

$$V_{O(dc)} = \frac{V_m}{\pi}(\cos\alpha - \cos\pi)$$

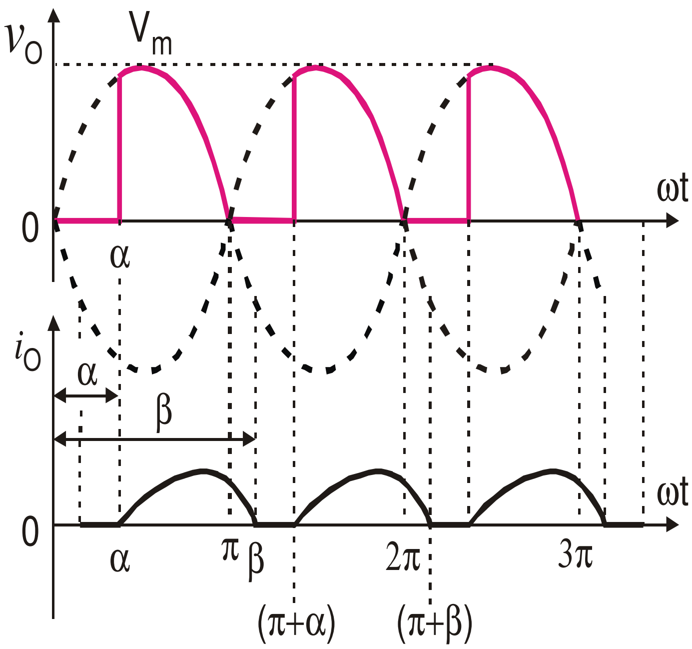
$$V_{O(dc)} = \frac{V_m}{\pi}(\cos\alpha - (-1))$$

$$V_{O(dc)} = \frac{V_m}{\pi}(1 + \cos\alpha); \text{ for R load, when } \beta = \pi$$

(iii) To calculate the RMS output voltage we use the expression

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi}\left[\int_{\alpha}^{\beta} V_m^2 \sin^2 \omega t . d(\omega t)\right]}$$

(iv) Discontinuous Load Current Operation with FWD



Thyristor T_1 is triggered at $\omega t = \alpha$;

T_1 conducts from $\omega t = \alpha$ to π

Thyristor T_2 is triggered at $\omega t = (\pi + \alpha)$;

T_2 conducts from $\omega t = (\pi + \alpha)$ to 2π

FWD conducts from $\omega t = \pi$ to β &

$v_O \approx 0$ during discontinuous load current.

(v) To Derive an Expression for the DC Output Voltage for a Single Phase Full Wave Controlled Rectifier with RL Load & FWD

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t = 0}^{\pi} v_O . d(\omega t)$$

$$\therefore \; V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t . d(\omega t)$$

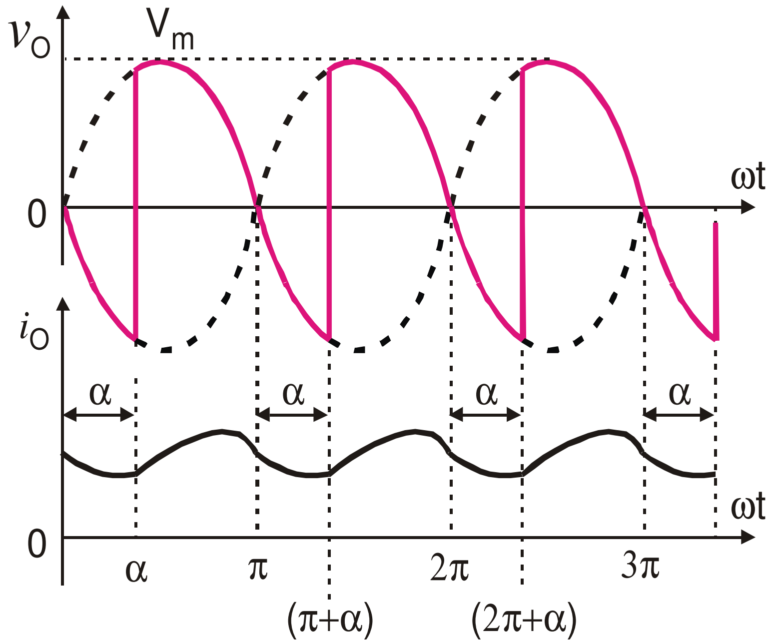
$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[-\cos \omega t \Big/_{\alpha}^{\pi} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[-\cos \pi + \cos \alpha\right] \; ; \; \cos \pi = -1$$

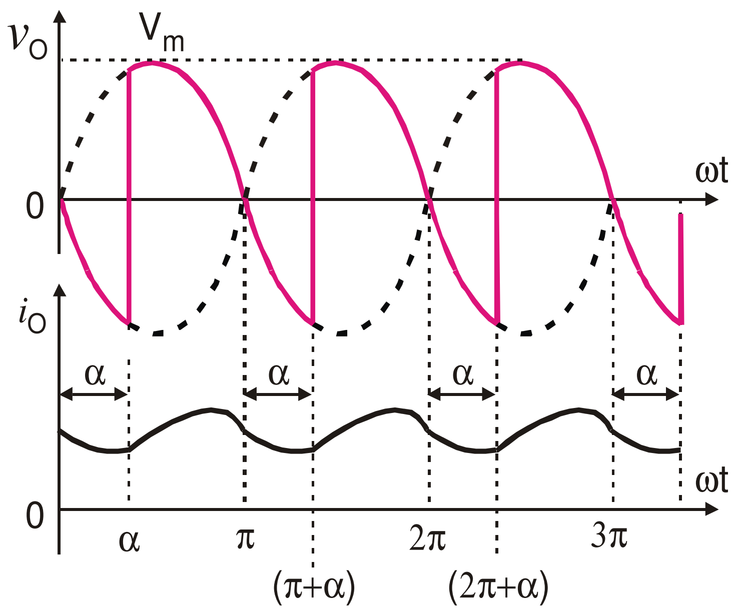
$$\therefore \; V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}(1 + \cos \alpha)$$

- The load current is discontinuous for low values of load inductance and for large values of trigger angles.
- For large values of load inductance the load current flows continuously without falling to zero.
- Generally the load current is continuous for large load inductance and for low trigger angles.

4.6.2 Continuous Load Current Operation (Without FWD)



(i) To Derive an Expression for Average / DC Output Voltage of Single Phase Full Wave Controlled Rectifier for Continuous Current Operation without FWD



$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=\alpha}^{(\pi+\alpha)} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{(\pi+\alpha)} V_m \sin \omega t . d(\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{(\pi+\alpha)} \right]$$

$$V_{O(dc)} = V_{dc}$$

$$= \frac{V_m}{\pi} \left[\cos\alpha - \cos(\pi+\alpha) \right] ;$$

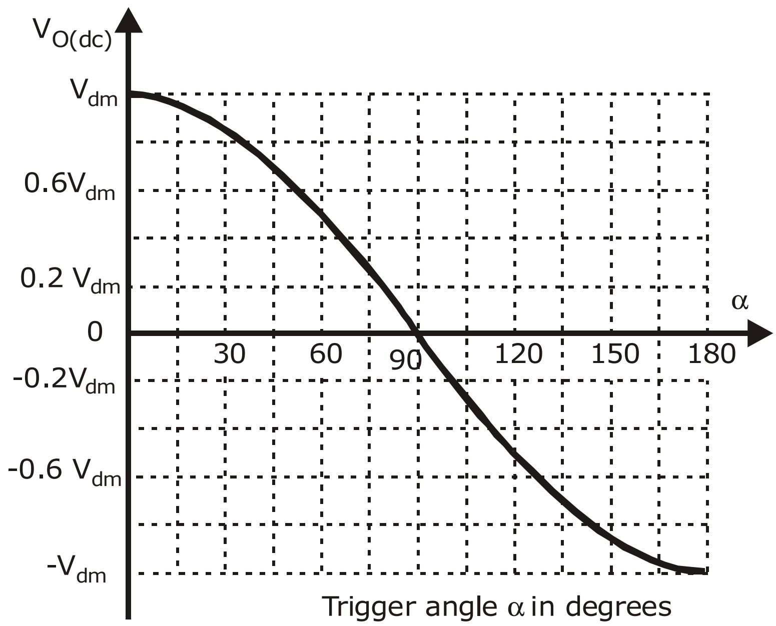
$$\cos(\pi+\alpha) = -\cos\alpha$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} [\cos\alpha + \cos\alpha]$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi} \cos\alpha$$

- By plotting *VO(dc) versus* α, we obtain the control characteristic of a single phase full wave controlled rectifier with RL load for continuous load current operation without FWD

Trigger angle α in degrees	$V_{O(dc)}$	**Remarks**
0	$V_{dm} = \left(\frac{2V_m}{\pi}\right)$	Maximum dc output voltage $V_{dc(\max)} = V_{dm} = \left(\frac{2V_m}{\pi}\right)$
30^0	$0.866\ V_{dm}$	$V_{dc} = V_{dm} \times \cos\alpha$
60^0	$0.5\ V_{dm}$	
90^0	$0\ V_{dm}$	
120^0	$-0.5\ V_{dm}$	
150^0	$-0.866\ V_{dm}$	
180^0	$-V_{dm} = -\left(\frac{2V_m}{\pi}\right)$	



By varying the trigger angle we can vary the output dc voltage across the load. Hence we can control the dc output power flow to the load.

For trigger angle α, 0 to 90^0 $\left(i.e.,\ 0 \le \alpha \le 90^0\right)$;

$\cos\alpha$ is positive and hence V_{dc} is positive

V_{dc} & I_{dc} are positive ; $P_{dc} = \left(V_{dc} \times I_{dc}\right)$ is positive

Converter operates as a Controlled Rectifier.

Power flow is from the ac source to the load.

For trigger angle α, 90^0 to 180^0

$$\left(i.e.,\ 90^0 \le \alpha \le 180^0\right),$$

$\cos\alpha$ is negative and hence

V_{dc} is negative; I_{dc} is positive ;

$$P_{dc} = \left(V_{dc} \times I_{dc}\right) \text{ is negative.}$$

In this case the converter operates as a Line Commutated Inverter.

Power flows from the load ckt. to the i/p ac source.

The inductive load energy is fed back to the i/p source.

Drawbacks of Full Wave Controlled Rectifier with Centre Tapped Transformer

- We require a centre tapped transformer which is quite heavier and bulky.
- Cost of the transformer is higher for the required dc output voltage & output power.
- Hence full wave bridge converters are preferred.

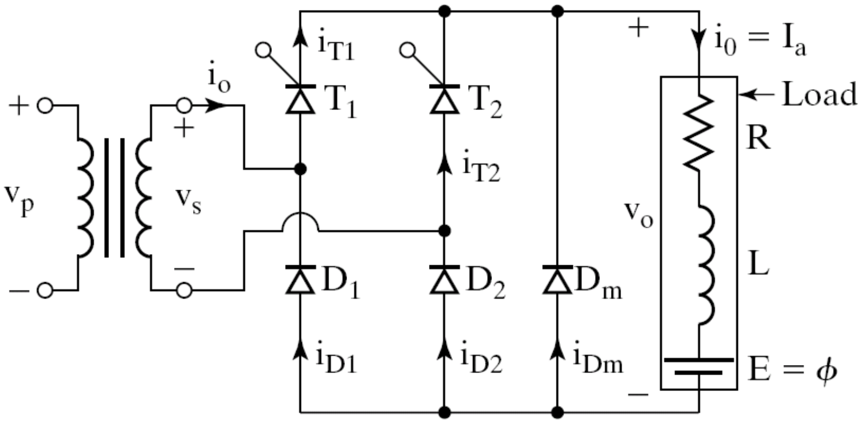
4.7 Single Phase Full Wave Bridge Controlled Rectifier

2 types of FW Bridge Controlled Rectifiers are

- Half Controlled Bridge Converter (Semi-Converter)
- Fully Controlled Bridge Converter (Full Converter)

The bridge full wave controlled rectifier does not require a centre tapped transformer

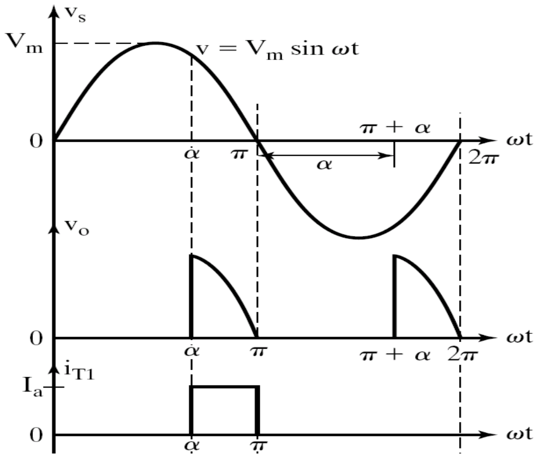
4.7.1 Single Phase Full Wave Half Controlled Bridge Converter (Single Phase Semi Converter)

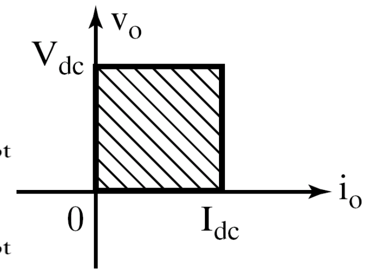


Trigger Pattern of Thyristors

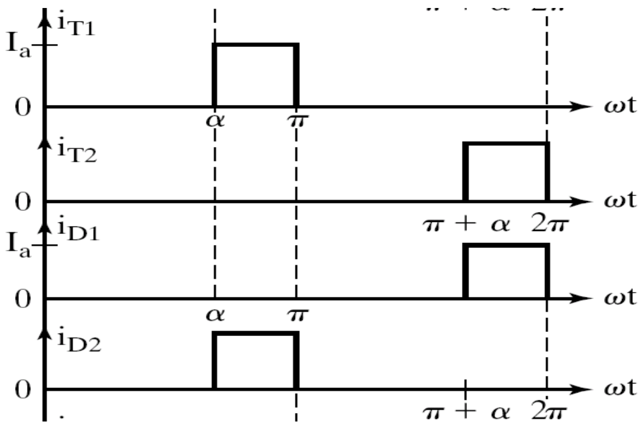
$$\text{Thyristor } T_1 \text{ is triggered at}$$
$$\omega t = \alpha, \text{ at } \omega t = (2\pi + \alpha), ...$$
$$\text{Thyristor } T_2 \text{ is triggered at}$$
$$\omega t = (\pi + \alpha), \text{ at } \omega t = (3\pi + \alpha), ...$$
$$\text{The time delay between the gating}$$
$$\text{signals of } T_1 \ \& \ T_2 = \pi \text{ radians or } 180^0$$

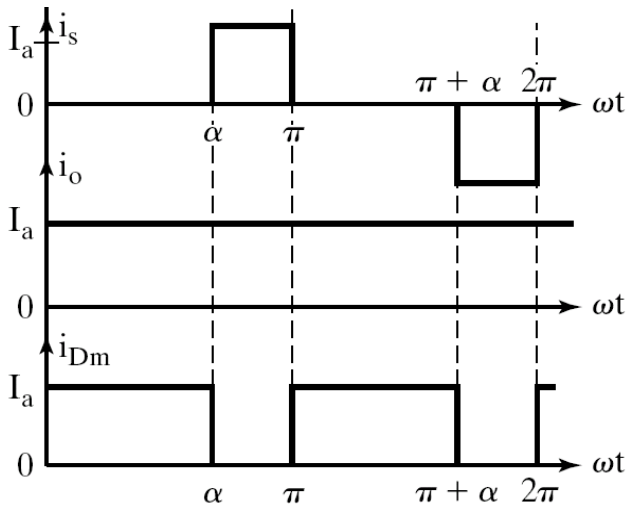
Waveforms of single phase semi-converter with general load & FWD for α > 900





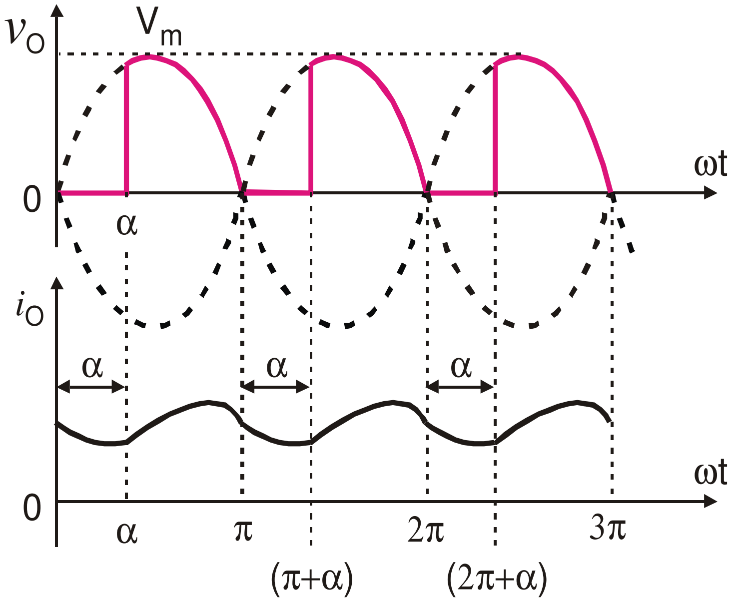
Single Quadrant Operation





Thyristor T_1 and D_1 conduct from $\omega t = \alpha$ to π
Thyristor T_2 and D_2 conduct from $\omega t = (\pi + \alpha)$ to 2π
FWD conducts during $\omega t = 0$ to α, π to $(\pi + \alpha)$, …..

Load Voltage & Load Current Waveform of Single Phase Semi Converter for $\alpha < 90^0$ & Continuous load current operation



(i) To Derive an Expression for The DC Output Voltage of A Single Phase Semi Converter with R, L, & E Load & FWD For Continuous, Ripple Free Load Current Operation

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=0}^{\pi} v_O . d(\omega t)$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\pi} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \pi + \cos \alpha \right] \;\; ; \; \cos \pi = -1$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha)$$

V_{dc} can be varied from a max.

value of $\dfrac{2V_m}{\pi}$ *to* 0 by varying α from 0 to π.

For $\alpha = 0$, The max. dc o/p voltage obtained is

$$V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi}$$

Normalized dc o/p voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dn}} = \frac{\frac{V_m}{\pi}(1+\cos\alpha)}{\left(\frac{2V_m}{\pi}\right)} = \frac{1}{2}(1+\cos\alpha)$$

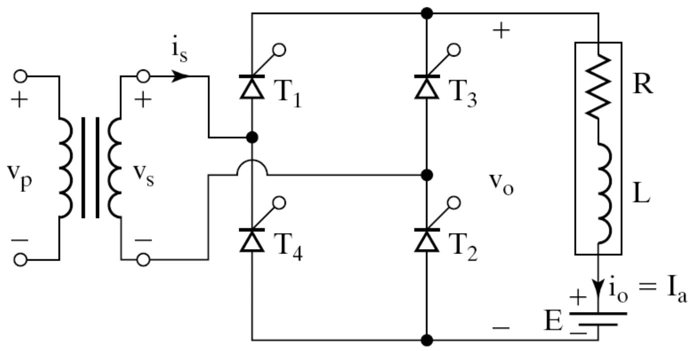
(ii) RMS O/P Voltage $V_{O(RMS)}$

$$V_{O(RMS)} = \left[\frac{2}{2\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t . d(\omega t)\right]^{\frac{1}{2}}$$

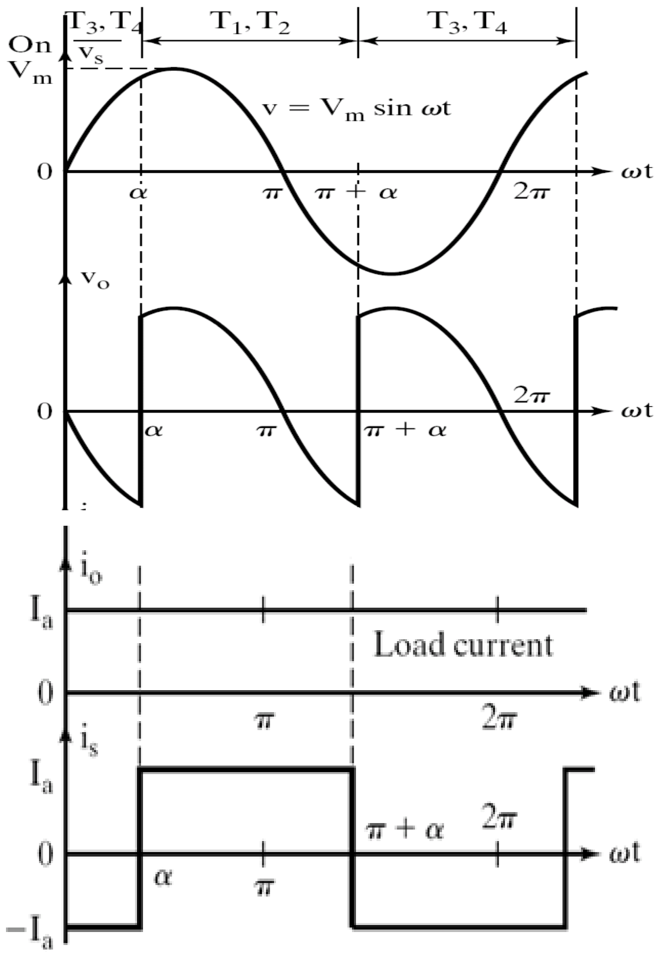
$$V_{O(RMS)} = \left[\frac{V_m^2}{2\pi}\int_{\alpha}^{\pi} (1-\cos 2\omega t) . d(\omega t)\right]^{\frac{1}{2}}$$

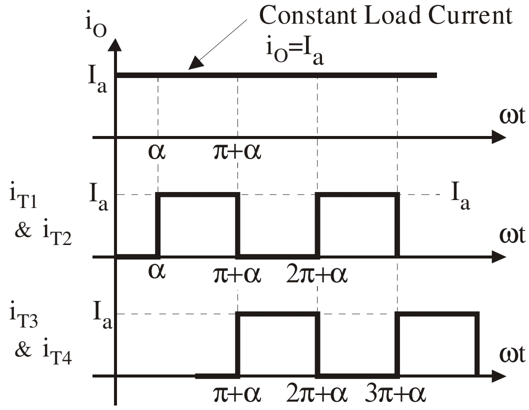
$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

4.7.2 Single Phase Full Wave Full Converter (Fully Controlled Bridge Converter) With R, L, & E Load



Waveforms of Single Phase Full Converter Assuming Continuous (Constant Load Current) & Ripple Free Load Current.





(i) To Derive An Expression For The Average DC Output Voltage of a Single Phase Full Converter assuming Continuous & Constant Load Current

The average dc output voltage can be determined by using the expression

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\left[\int_0^{2\pi} v_O.d(\omega t)\right];$$

The o/p voltage waveform consists of two o/p pulses during the input supply time period of 0 to 2π radians. Hence the Average or dc o/p voltage can be calculated as

$$V_{O(dc)} = V_{dc} = \frac{2}{2\pi}\left[\int_\alpha^{\pi+\alpha} V_m \sin\omega t.d(\omega t)\right]$$

$$V_{O(dc)} = V_{dc} = \frac{2V_m}{2\pi}\left[-\cos\omega t\right]_\alpha^{\pi+\alpha}$$

$$V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

Maximum average dc output voltage is calculated for a trigger angle $\alpha = 0^0$ and is obtained as

$$V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi}\times\cos(0) = \frac{2V_m}{\pi}$$

$$\therefore V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi}$$

The normalized average output voltage is given by

$$V_{dcn} = V_n = \frac{V_{O(dc)}}{V_{dc(\max)}} = \frac{V_{dc}}{V_{dm}}$$

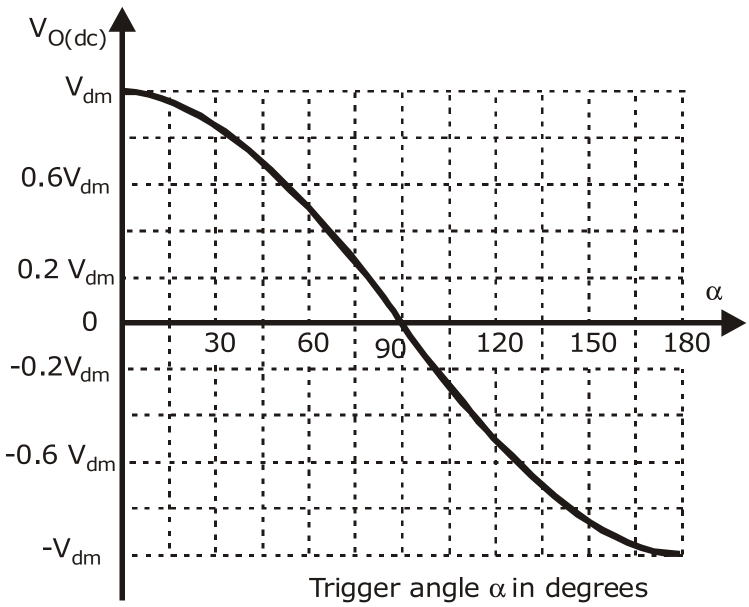
$$\therefore V_{dcn} = V_n = \frac{\frac{2V_m}{\pi}\cos\alpha}{\frac{2V_m}{\pi}} = \cos\alpha$$

By plotting *VO(dc) versus* α, we obtain the control characteristic of a single phase full wave fully controlled bridge converter (single phase full converter) for constant & continuous load current operation.

To plot the control characteristic of a Single Phase Full Converter for constant & continuous load current operation.
We use the equation for the average/ dc output voltage

$$V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

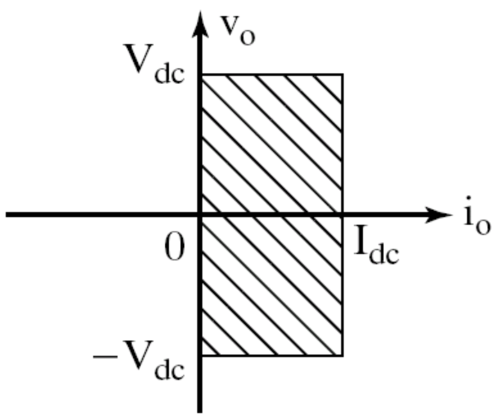
Trigger angle α in degrees	$V_{O(dc)}$	Remarks
0	$V_{dm} = \left(\frac{2V_m}{\pi}\right)$	Maximum dc output voltage $V_{dc(\max)} = V_{dm} = \left(\frac{2V_m}{\pi}\right)$
30^0	$0.866\ V_{dm}$	
60^0	$0.5\ V_{dm}$	
90^0	$0\ V_{dm}$	
120^0	$-0.5\ V_{dm}$	
150^0	$-0.866\ V_{dm}$	
180^0	$-V_{dm} = -\left(\frac{2V_m}{\pi}\right)$	



- During the period from $\omega t = \alpha$ to π the input voltage vS and the input current iS are both positive and the power flows from the supply to the load.
- The converter is said to be operated in the rectification mode Controlled Rectifier Operation for $0 < \alpha < 900$
- During the period from $\omega t = \pi$ to $(\pi+\alpha)$, the input voltage vS is negative and the input current iS is positive and the output power becomes negative and there will be reverse power flow from the load circuit to the supply.
- The converter is said to be operated in the inversion mode.

Line Commutated Inverter Operation for $900 < \alpha < 1800$

Two Quadrant Operation of a Single Phase Full Converter



(ii) To Derive an Expression for the RMS Value of the Output Voltage

The rms value of the output voltage is calculated as

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_{0}^{2\pi} v_O^2 . d(\omega t)\right]}$$

The single phase full converter gives two output voltage pulses during the input supply time period and hence the single phase full converter is referred to as a two pulse converter. The rms output voltage can be calculated as

$$V_{O(RMS)} = \sqrt{\frac{2}{2\pi}\left[\int_{\alpha}^{\pi+\alpha} v_O^2 . d(\omega t)\right]}$$

The single phase full converter gives two output voltage pulses during the input supply time period and hence the single phase full converter is referred to as a two pulse converter. The rms output voltage can be calculated as

$$V_{O(RMS)} = \sqrt{\frac{2}{2\pi}\left[\int_{\alpha}^{\pi+\alpha} v_O^2 . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi}\left[\int_{\alpha}^{\pi+\alpha} V_m^2 \sin^2 \omega t . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi}\left[\int_{\alpha}^{\pi+\alpha} \sin^2 \omega t . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi}\left[\int_{\alpha}^{\pi+\alpha} \frac{(1-\cos 2\omega t)}{2} . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[\int_{\alpha}^{\pi+\alpha} d(\omega t) - \int_{\alpha}^{\pi+\alpha} \cos 2\omega t . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi}\left[\int_{\alpha}^{\pi+\alpha} V_m^2 \sin^2 \omega t . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi}\left[\int_{\alpha}^{\pi+\alpha} \sin^2 \omega t . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi}\left[\int_{\alpha}^{\pi+\alpha} \frac{(1-\cos 2\omega t)}{2} . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[\int_{\alpha}^{\pi+\alpha} d(\omega t) - \int_{\alpha}^{\pi+\alpha} \cos 2\omega t . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\omega t)\Big/_{\alpha}^{\pi+\alpha} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\pi+\alpha}\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\pi+\alpha-\alpha) - \left(\frac{\sin 2(\pi+\alpha) - \sin 2\alpha}{2}\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\pi) - \left(\frac{\sin(2\pi+2\alpha) - \sin 2\alpha}{2}\right)\right]};$$

$$\sin(2\pi+2\alpha) = \sin 2\alpha$$

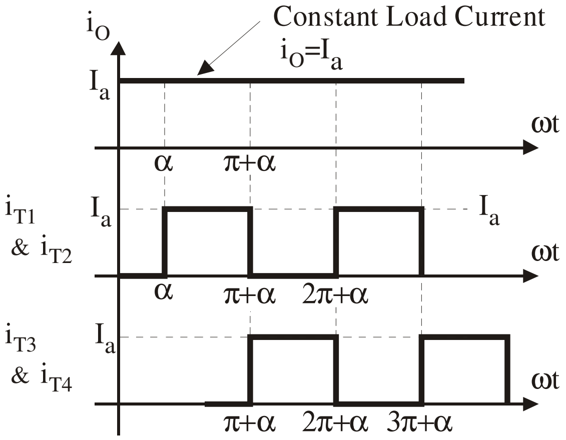
$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\pi) - \left(\frac{\sin 2\alpha - \sin 2\alpha}{2}\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}(\pi) - 0} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

$$\therefore V_{O(RMS)} = \frac{V_m}{\sqrt{2}} = V_S$$

Hence the rms output voltage is same as the rms input supply voltage

4.7.3 Thyristor Current Waveforms



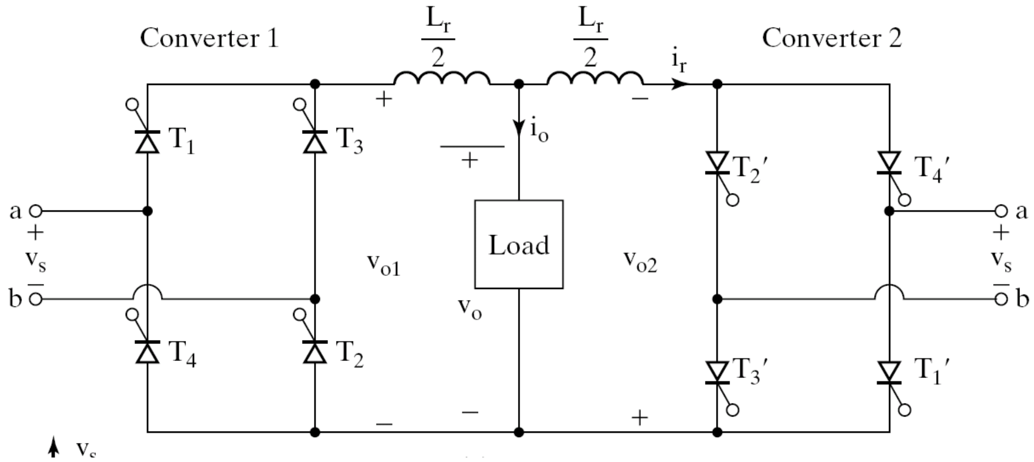
The rms thyristor current can be calculated as

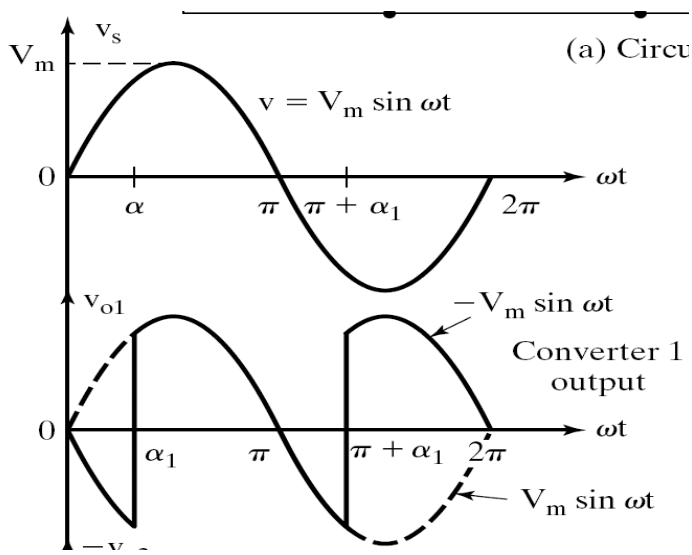
$$I_{T(RMS)} = \frac{I_{O(RMS)}}{\sqrt{2}}$$

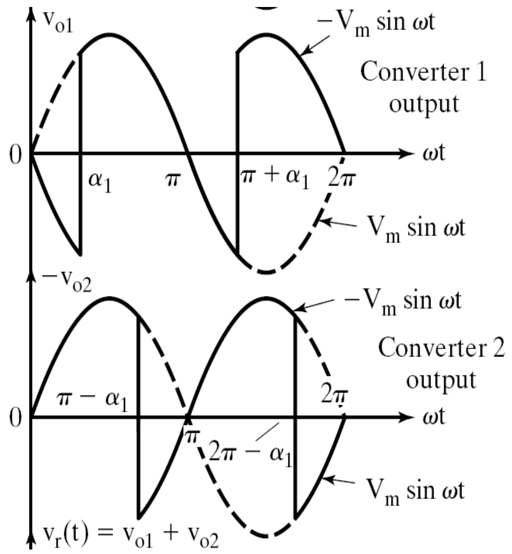
The average thyristor current can be calculated as

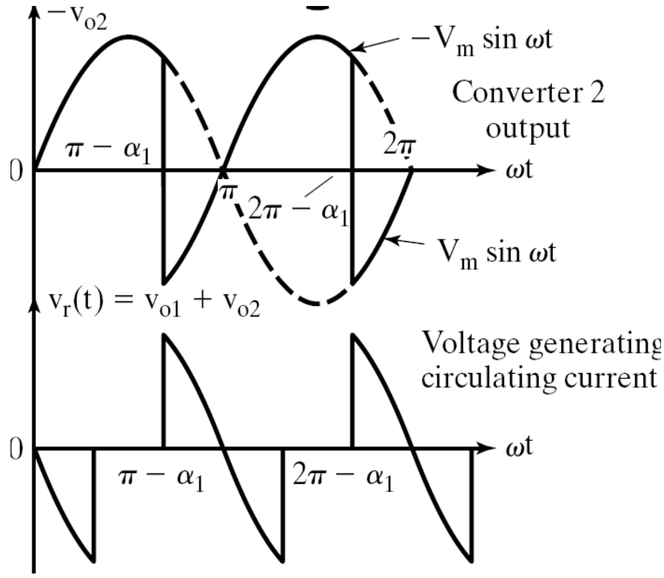
$$I_{T(Avg)} = \frac{I_{O(dc)}}{2}$$

4.8 Single Phase Dual Converter









The average dc output voltage of converter 1 is

$$V_{dc1} = \frac{2V_m}{\pi}\cos\alpha_1$$

The average dc output voltage of converter 2 is

$$V_{dc2} = \frac{2V_m}{\pi}\cos\alpha_2$$

In the dual converter operation one converter is operated as a controlled rectifier with $\alpha < 90^0$ & the second converter is operated as a line commutated inverter in the inversion mode with $\alpha > 90^0$

$$\therefore \quad V_{dc1} = -V_{dc2}$$

$$\frac{2V_m}{\pi}\cos\alpha_1 = \frac{-2V_m}{\pi}\cos\alpha_2 = \frac{2V_m}{\pi}(-\cos\alpha_2)$$

$\therefore \quad \cos\alpha_1 = -\cos\alpha_2$

or

$\cos\alpha_2 = -\cos\alpha_1 = \cos(\pi-\alpha_1)$

$\therefore \quad \alpha_2 = (\pi-\alpha_1)$ or

$(\alpha_1+\alpha_2) = \pi$ radians

Which gives

$\alpha_2 = (\pi-\alpha_1)$

(i) To Obtain an Expression for the Instantaneous Circulating Current

- v_{O1} = Instantaneous o/p voltage of converter 1.
- v_{O2} = Instantaneous o/p voltage of converter 2.
- The circulating current *ir* can be determined by integrating the instantaneous voltage difference (which is the voltage drop across the circulating current reactor *Lr*), starting from $\omega t = (2\pi - \alpha 1)$.
- As the two average output voltages during the interval $\omega t = (\pi+\alpha 1)$ to $(2\pi - \alpha 1)$ are equal and opposite their contribution to the instantaneous circulating current *ir* is zero.

$$i_r = \frac{1}{\omega L_r}\left[\int_{(2\pi-\alpha_1)}^{\omega t} v_r . d(\omega t)\right]; \quad v_r = (v_{O1} - v_{O2})$$

As the o/p voltage v_{O2} is negative

$$v_r = (v_{O1} + v_{O2})$$

$$\therefore \quad i_r = \frac{1}{\omega L_r}\left[\int_{(2\pi-\alpha_1)}^{\omega t} (v_{O1} + v_{O2}) . d(\omega t)\right];$$

$$v_{O1} = -V_m \sin\omega t \text{ for } (2\pi-\alpha_1) \text{ to } \omega t$$

$$i_r = \frac{V_m}{\omega L_r}\left[\int_{(2\pi-\alpha_1)}^{\omega t} -\sin\omega t . d(\omega t) - \int_{(2\pi-\alpha_1)}^{\omega t} \sin\omega t . d(\omega t)\right]$$

$$i_r = \frac{2V_m}{\omega L_r}(\cos\omega t - \cos\alpha_1)$$

The instantaneous value of the circulating current depends on the delay angle.

For trigger angle (delay angle) $\alpha_1 = 0$, the magnitude of circulating current becomes min. when $\omega t = n\pi,\ n = 0, 2, 4, ...$ & magnitude becomes max. when $\omega t = n\pi,\ n = 1, 3, 5, ...$

If the peak load current is I_p, one of the converters that controls the power flow may carry a peak current of where

$$I_p = I_{L(\max)} = \frac{V_m}{R_L},$$

&

$$i_{r(\max)} = \frac{4V_m}{\omega L_r} = \text{ max. circulating current}$$

The Dual Converter Can Be Operated In Two Different Modes Of Operation

- Non-circulating current (circulating current free) mode of operation.
- Circulating current mode of operation

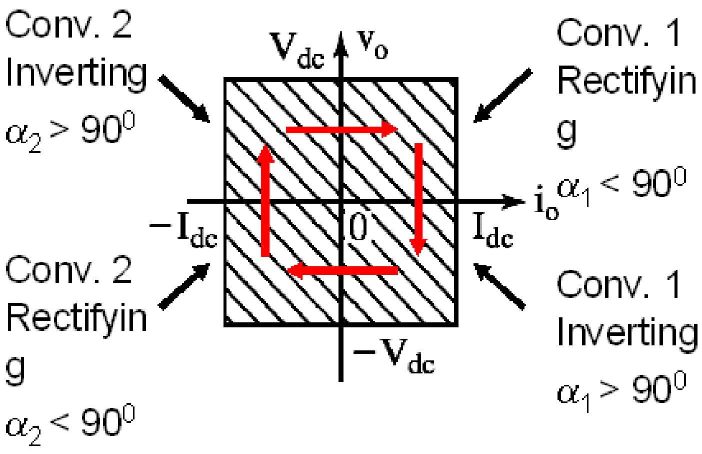
Non-Circulating Current Mode of Operation

- In this mode only one converter is operated at a time.
- When converter 1 is ON, 0 < α1 < 900
- V_{dc} is positive and I_{dc} is positive.
- When converter 2 is ON, 0 < α2 < 900
- V_{dc} is negative and I_{dc} is negative.

Circulating Current Mode Of Operation

- In this mode, both the converters are switched ON and operated at the same time.
- The trigger angles α1 and α2 are adjusted such that (α1 + α2) = 1800 ; α2 = (1800 - α1).
- When 0 <α1 <900, converter 1 operates as a controlled rectifier and converter 2 operates as an inverter with 900 <α2<1800.
- In this case V_{dc} and I_{dc}, both are positive.
- When 900 <α1 <1800, converter 1 operates as an Inverter and converter 2 operated as a controlled rectifier by adjusting its trigger angle α2 such that 0 <α2<900.
- In this case Vdc and Idc, both are negative.

4.8.1 Four Quadrant Operation



Advantages of Circulating Current Mode of Operation

- The circulating current maintains continuous conduction of both the converters over the complete control range, independent of the load.
- One converter always operates as a rectifier and the other converter operates as an inverter, the power flow in either direction at any time is possible.
- As both the converters are in continuous conduction we obtain faster dynamic response. i.e., the time response for changing from one quadrant operation to another is faster.

Disadvantages of Circulating Current Mode of Operation

- There is always a circulating current flowing between the converters.
- When the load current falls to zero, there will be a circulating current flowing between the converters so we need to connect circulating current reactors in order to limit the peak circulating current to safe level.
- The converter thyristors should be rated to carry a peak current much greater than the peak load current.

Recommended questions:

1. Give the classification of converters, based on: a) Quadrant operation b) Number of current pulse c) supply input. Give examples in each case.
2. With neat circuit diagram and wave forms, explain the working of 1 phase HWR using SCR for R-load. Derive the expressions for V_{dc} and I_{dc}.
3. With a neat circuit diagram and waveforms, explain the working of 1-phase HCB for R-load and R-L-load.
4. Determine the performance factors for 1-phase HCB circuit.

5. With a neat circuit diagram and waveforms, explain the working of 1-phase FCB for R and R-L-loads.
6. Determine the performance factors for 1-phase FCB circuit.
7. What is dual converter? Explain the working principle of 1-phase dual converter. What are the modes of operation of dual converters? Explain briefly.
8. With a neat circuit diagram and waveforms explain the working of 3 phase HHCB using SCRs. Obtain the expressions for V_{dc} and I_{dc}.
9. With a neat circuit diagram and waveforms, explain the working of 3-phase HWR using SCRs. Obtain the expressions for V_{dc} and $I_{dc.}$
10. With a neat circuit diagram and waveforms, explain the working of 3 phase FCB using SCRs. Obtain the expressions for V_{dc} and I_{dc}.
11. Draw the circuit diagram of 3 phase dual converter. Explain its working?
12. List the applications of converters. Explain the effect of battery in the R-L-E load in converters.
13. A single phase half wave converter is operated from a 120V, 60 Hz supply. If the load resistive load is R=10Ω and the delay angle is α=π/3, determine a) the efficiency b) the form factor c) the transformer utilization factor and d) the peak inverse voltage (PIV) of thyristor T1
14. A single phase half wave converter is operated from a 120 V, 60 Hz supply and the load resistive load is R=10□. If the average output voltage is 25% of the maximum possible average output voltage, calculate a) the delay angel b) the rms and average output current c) the average and ram thyristor current and d) the input power factor.
15. A single half wave converter is operated from a 120 V, 60Hz supply and freewheeling diodes is connected across the load. The load consists of series-connected resistance R=10Ω, L=mH, and battery voltage E=20V. a) Express the instantaneous output voltage in a Fourier series, and b) determine the rms value of the lowest order output harmonic current.
16. A single phase semi-converter is operated from 120V, 60 Hz supply. The load current with an average value of Ia is continuous with negligible ripple content. The turns ratio of the transformer is unity. If the delay angle is A= π/3, calculate a) the harmonic factor of input current b) the displacement factor and c) the input power factor.
17. A single phase semi converter is operated from 120V, 60Hz supply. The load consists of series connected resistance R=10Ω, L=5mH and battery voltage E=20V. a) Express the instantaneous output voltage in a Fourier series, b) Determine the rms value of the lowest order output harmonic current.
18. The three phase half wave converter is operated from a three phase Y connected 220V, 60Hz supply and freewheeling diodes is connected across the load. The load consists of series connected resistance R=10Ω, L=5mH and battery voltage E=120V. a) Express the instantaneous output voltage in a Fourier series and b) Determine the rms value of the lowest order output harmonic current.

13. without a series inductor? What is the ratio of peak resonant to load current for resonant pulse commutation that would minimize the commutation losses?
14. Why does the commutation capacitor in a resonant pulse commutation get over charged?
15. How is the voltage of the commutation capacitor reversed in a commutation circuit?
16. What is the type of a capacitor used in high frequency switching circuits?